SKAGIT RIVER
DAM FAILURE INUNDATION STUDY
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1.0 SUMMARY

In accordance with Federal 'Regulations Governing Safety of Water Power Projects and Project Works', the Federal Energy Regulatory Commission (FERC) requires an applicant or licensee to file an 'Emergency Action Plan' (EAP), as appropriate, for proposed or existing hydropower generation projects (FERC Order #122, January 21, 1981). One part of the preparation of an EAP is the analysis of the potential downstream inundation resulting from a dam failure.

This report includes a discussion of assumed modes of failure and inundation analyses for Ross Dam, Diablo Dam, and Gorge Dam which are owned by the City of Seattle, City Light Department and comprise the Skagit River Project (FERC PROJECT 533). **The causes of failure assumed in this study are "worst case" hypothetical conditions that are assumed for the purpose of emergency action planning.**

This report includes analyses of the inundations that result from the following failure cases:

- A Failure of Ross Dam which is followed by failures of the Diablo and Gorge Dams by overtopping.
- A Failure of Diablo Dam followed by failure of Gorge Dam by overtopping.
- A Failure of Gorge Dam.

For the inundations that result from these cases, the maximum discharge and the elapsed times from the failure to flood stage and to the maximum discharge are given. Maximum discharges are shown in tables and figures for several points on the Skagit River.
The technical analysis used to study the inundation is "full equations routing" which predicts flood wave movement using the complete energy and momentum equations. Full equations routing, rather than conventional hydrologic or kinematic wave routing, is needed for inundation mapping because failure assumptions cause rapidly changing stages that alter flood wave motion.

The inundation studies show that very large flows would result in the Skagit Valley from the assumed failure of Ross, Diablo and Gorge Dams. Failures of Diablo and Gorge cause less severe flooding. The failure of Gorge Dam alone causes flooding only immediately below the dam. The maximum discharges for each case are shown in Table 1.1. Note that for the Gorge Dam failure the increase in the maximum discharge at Concrete is caused by the increase in natural or "base" river flows as the drainage area of the watershed increases.
TABLE 1-1
Simulation Results Downstream of Skagit River Project

<table>
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<th>FAILURE CASE</th>
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<tr>
<td></td>
<td>Diablo Initial Flow</td>
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<tr>
<td></td>
<td>22,000 cfs</td>
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<td></td>
<td>Peak Flow (cfs)</td>
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<td></td>
<td>Elapsed Flow (hours)</td>
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<td>Gorge Dam Only</td>
<td>22,000</td>
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<td></td>
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<tr>
<td>Diablo Dam followed by Overtopping Failure of</td>
<td>2,530,000</td>
</tr>
<tr>
<td>Gorge Dam</td>
<td>0.1</td>
</tr>
<tr>
<td>Ross Dam Followed by Overtopping Failures of</td>
<td>5,870,000</td>
</tr>
<tr>
<td>Diablo and Gorge Dams</td>
<td>0.4</td>
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The flow produced by the failure of Ross Dam is several times larger than the maximum observed historic flood at Sedro-Woolley (220,000 cfs, November 30, 1909). At Sedro-Woolley the maximum historic flood produces a maximum water surface elevation of 50 feet, while the maximum dam failure discharge of 1,740,000 cfs produces a maximum stage of 75 feet at the same location. The flooding between Sedro-Woolley and Puget Sound from the Ross, Diablo and Gorge inundation case would be greater than that mapped by the Corps of Engineers for a standard Project Flood (Corps of Engineers, 1967). Levees protecting urban settlements and farm lands between Sedro-Woolley and Puget Sound would be expected to fail, and low lying urban centers like Burlington would be flooded.

However, the inundation flood in the lower Skagit Valley is not a sudden, high velocity "wall of water". It is more similar to a flood caused by rainfall or snowmelt, and it occurs 7 to 11 hours after the hypothetical catastrophe at Ross Dam. The significant results of the inundation analysis that are useful in planning warning systems, are the lengths of time before flooding occurs in downstream communities. For example, for the failure of Ross, Diablo and Gorge Dams, the elapsed time before the Skagit River reaches flood stage at Concrete is 3.5 hours. This "warning-time" would be very important in an actual emergency.
2.0 INTRODUCTION

This report summarizes Hydrocomp's study of the Skagit River Project (FERC Project #553), conducted for the Seattle City Light Department as part of their Emergency Action Plan (EAP) for the Federal Energy Regulatory Commission (FERC). The study included an inspection of the Skagit River System and an analysis of the movement of the flood waters following assumed failures of the Ross, Diablo and Gorge Dams. Results of the study do not in any way reflect upon the structural integrity of the dams, and are not to be construed as such. The dams are considered to fail in order to study the effects, only for the purpose of emergency action planning.

2.1 The Skagit River System

The Skagit River Project consists of 1) the Ross Development - Ross Dam, which is a 540 foot concrete gravity arch dam, 2) The Diablo Development - Diablo Dam, which is a 389 foot concrete arch dam, and 3) The Gorge Development - Gorge Dam, which is a 300 foot concrete gravity arch dam. All three are on the Skagit River. The reservoir behind Ross Dam is Ross Lake. Below Gorge Dam, the Skagit River flows in a narrow canyon for 15 miles to Marblemount. The Skagit River broadens at Marblemount and flows through a one to three mile wide alluvial valley for 50 miles. It is joined by the Cascade, Sauk, and Baker Rivers before reaching the wide, flat Skagit Delta, and ultimately Puget Sound. Figure 2-1 shows the Skagit system and the locations referred to in this report for discharges and water surface elevations.
Figure 2.1

Skagit River
Hydrocomp

x Indicates Location of Flow Data
Listed in Tables and Graphs
2.2 FERC Guidelines and Criteria

The requirements for EAP's established by the Federal Energy Regulatory Commission recognize that every dam runs at least some risk of failure. As a result, the owner of any water impounding structure whose failure could endanger life or property, is required by FERC order #122, Part 12, Subpart Emergency Action Plans, to prepare an EAP.

"In a nutshell, the FERC process is designed to find out what might happen, what problems would arise, what people or property would be affected, and what can be done about it" (Kopfler, 1980).

This study will aid the City of Seattle in their preparation of the EAP for the Skagit Project. In accordance with FERC guidelines the most probable modes and causes failures were identified, and the area affected by the largest of these failures was delineated.

2.3 Failure Causes

The causes of sudden releases of water from the Ross, Diablo or Gorge Developments are assumed to be massive earthquakes or landslides. Because the foundations for these dams are rock, the piping or erosive failures that are assumed for safety studies of earth dams are not pertinent for the Skagit River Project. The intent is to assume a "worst case" catastrophe, so partial failures of the dams are not considered. Partial failures or slowly developing failures are not considered appropriate for concrete arch dams, since any loss of structural integrity would result in a rapid collapse of most of the dam structure.
The failure modes assumed for the Skagit River dams are the same as were used in prior FERC studies of concrete arch and gravity dams (Hydrocomp, 1974 and 1981). These failure causes are:

- a sudden loss of structural integrity, most likely due to a massive earthquake in the Skagit River Valley, or

- landslides sufficient to cause excessive overflow depths (50 feet or more) at the dam sites, which are then assumed to result in a failure of the structure, or

- a failure at Ross or Diablo Dams, which results in excessive overflow depths at downstream dams, leading to an assumed failure.

For practical purposes, an earthquake induced failure or landslide-induced failure at a project have the same effect. In each case the dam is assumed to be removed over a 60 second timespan. In fact if a dam is removed in 30 or 60 seconds, or in 2, 5, or 10 minutes, the failure produces nearly the same discharges in the Skagit River Valley.

There are no identified spillway, foundation, abutment, or operational deficiencies at any of the dams in the Skagit Project. With no known unsafe conditions, the failure causes described above were found by asking the question, "If the dams were to suddenly fail, what would be the most likely cause?" Earthquakes and landslides on a massive scale sufficient to cause dam failures are in fact highly improbable. They are simply judged to be more likely than other catastrophies like meteor strikes.
2.4 Contents of the Report

Section 3.0 describes the Full Equations (FULEQ) software system used in this study and recounts the specific techniques and assumptions used to define the dam failure problem. The use of topographic data and the description of the system for use by the computer program are detailed. For those interested in the method of analysis used in FULEQ, a brief discussion of the flow routing process is given in Appendix A.

Section 4.0 is a discussion of modes of failure for the dams of the Skagit Project. The next section (5.0) presents the results of the computer simulations. The movement of the flood waves are described as they pass through the channel system. Section 6.0 contains the study conclusions.
3.0 FULEQ SOFTWARE AND THE SKAGIT RIVER SYSTEM

The principles of long wave motion in open channels and the method of characteristics have been used as the basis for Hydrocomp's FULEQ computer system. A discussion of the underlying principles used to construct the program, and of the assumptions made in FULEQ, is presented in Appendix A. The FULEQ system consists of 62 subprograms and 4 separate stand-alone programs, all written in FORTRAN and operating on Hydrocomp's HP3000 Computer System. FULEQ is a comprehensive model requiring an unusually detailed description of the channel system. The major part of the model deals solely with checking and interpreting the information used to describe the open channel system.

3.1 Model Concepts and Terminology

In order to impart to the computer a precise description of the channel system, a nomenclature and a set of conventions are developed. A channel system or network is represented by channels along which water flows, and by junctions where several channels meet. Channels are normally much longer than their depth or width, and so they may be thought of as being one-dimensional. The error introduced by assuming that the only direction of flow is parallel to the channel direction is small and may be corrected for quite easily. The items of interest are the depth of flow and the flowrate at each point in the channel. The depth of flow is taken as the maximum depth, perpendicular to the assumed bottom profile. The bottom profile is assumed because local irregularities are considered as part of the roughness and not part of the bottom profile. This assumption permits developing a smooth bottom profile.
Naturally occurring lakes, as well as man-made lakes, may be present in the network. There may be a variety of structures such as bridge piers, siphons, weirs, dams, pumps, turbines, points where water is taken out of the channel and points where water is discharged to the channel. All or part of the flow in a channel may enter a closed conduit and flow under pressure for some distance before it returns to an open channel. The network of channels and/or closed conduits must have clearly established boundaries or limits.

The limits of the channel network are established by conceptually cutting various channels or closed conduits so that the network is isolated from its surroundings. An external boundary point is established at each place cut. The behavior of the surrounding network at these points must be properly specified to analyze the isolated system. Naturally enough, the conditions required at the boundary points are called external boundary conditions or merely boundary conditions.

These external boundary conditions are of two types: forced and free. A forced boundary condition imposes either a flowrate or depth as a function of time at the boundary point. The solution must honor the given value of depth or flowrate. An example of a forced boundary condition is the fluctuation of the water surface caused by tides at the mouth of a river discharging into the ocean. A free boundary condition specifies a relationship between the depth and flowrate at the boundary point. Neither the depth nor the flowrate is forced. The free boundary condition is established in one of three ways. (1) A control structure may provide the relationship, or (2) the flow in the channel may naturally pass through critical depth near the boundary point, and so a relationship may be assumed between depth and flowrate given
by the critical flow in the channel. (3) Frequently neither a control structure nor a critical depth section is available at a downstream boundary point. The relationship between flow and depth must then be developed by other means.

3.1.1 Exterior and Interior Nodes
The representation of a complex channel network, so that the depth and flowrate as a function of location and time can be determined, is the next task. A cursory analysis of the physical principles governing the flow shows that it is impossible to develop explicit solutions for depth and flowrate in a general network. We must be content with developing approximate solutions at selected points along the channels and at selected points in time. Points along the channel where a computation of the values of depth and flowrate is needed are called nodes. There are two classes of nodes: exterior nodes and interior nodes. These classes can only be defined when the concepts of a branch and an interaction point have been developed.

An interaction point is best defined by giving a series of examples. A junction is an interaction point because flow from one channel entering the junction may influence the flow characteristics in the other channels entering the junction. Interaction points also occur whenever there is a fixed relationship established between flowrate and depth of flow in the channel. An example would be the outlet works of a dam controlling flow past the dam. An interaction point also occurs whenever the point inflow or point outflow from the channel depends on values of depth or flowrate or both in the channel. An example of this interaction point would be a side-channel spillway. Finally, interaction points are created at external boundary points. Each branch will have a node at each end and
perhaps other nodes distributed over its length. The nodes at each end are called exterior nodes, and the remaining nodes on the branch are called interior nodes. A branch must always have exterior nodes but does not have to have interior nodes.

3.1.2 Branches
The concept of a branch is very significant because every branch is similar. A branch is simply a reach of channel which has been artificially separated from the rest of the channel, in order to study it. No special conditions exist for a branch and therefore the system of equations written to describe the relationships among the depths and flowrates applies for every branch. Only the number of equations will vary, with \((2n-2)\) equations for a branch with \(n\) nodes on it. The requirement for this similarity between branches implies that sufficient interaction points are defined so that the channels connecting these interaction points are in fact similar. This requirement can be used to define interaction points in terms of branches.

The relationship between exterior nodes and interaction points needs explaining. Each interaction point must involve one or more exterior nodes where at least one exterior node is on a branch. Exterior nodes serve as a linkage between the relationships which exist among the variables at an interaction point and the variables at the interior nodes of the branches connected to the interaction point. The exterior nodes are viewed as belonging to both the interaction points and to the branches. It is possible to solve for the values of depth and flowrate at the exterior nodes without knowing the values of depth and flowrate at the interior nodes.
The distinction made between interior and exterior nodes is motivated by this possibility. In fact, the model user will specify the relationships among exterior nodes in an exterior node matrix. This specification will give the relationship between exterior nodes and interaction points and indirectly the relationship between branches. This approach gives the maximum flexibility without unduly complicating the programming.

3.1.3 Control Structures

The description of control structures is the most troublesome aspect of the system. A control structure has the following general characteristics:

1. The device has an input and an output side. An arbitrary distinction is imposed if a natural distinction does not exist. The flow through the device is treated as positive if its direction is from the input side to the output side.

2. The control structure is assumed to be sufficiently localized so that neither changes in storage nor changes in momentum within the control structure are significant. As a result, the relationships for a control structure are steady state relationships with the flowrate at both input and output sides being equal at all times.

3. The hydraulic characteristics of the device are determined in one or more of the following ways:

   a. depth at the input side (velocity may be needed in some cases)
b. depth at both input and output sides (velocities may be needed in some cases)

c. a variable setting of the device geometry.

No other details of the control structure are considered to be significant. The input and output sides of a control structure are normally associated with distinct exterior nodes. That is, a control structure connects, at least in a conceptual sense, two and only two exterior nodes. The depths and/or flowrates required by the control structure are derived from the flow conditions at the exterior nodes of the control structure. The flow through a control structure is uniquely identified by requiring the flow at one of the exterior nodes of the control structure to be the same as the flow through the control structure.

3.2 The Skagit River System

In applying the nomenclature developed for FULEQ to the project setting, our channel network consists of the Skagit River divided into eight branches; three control structures, representing the dams; and 193 interior nodes at 500 to 5000 foot intervals. The eight branches were defined as follows:

1) Ross Lake - Length equals 23 miles, divided into 45 interior nodes. Downstream control is a stage vs. discharge control structure, upstream control is a discharge vs. time curve.
2) Diablo Lake - Length equals 4 miles, divided into 23 interior nodes. Downstream control is a stage vs. discharge control structure, upstream control is the discharge vs. time curve of the outflow from Branch 1 (Ross Dam).

3) Gorge Lake - Length equals 5 miles, divided into 12 interior nodes. Downstream control is a stage vs. discharge control structure, upstream control is the discharge vs. time curve of the outflow from Diablo Dam.

4) Skagit River from Gorge Dam to Newhalem - Length equals three miles, divided into 13 interior nodes. Downstream control is provided by an equality of elevation and discharge with the upstream Exterior node for Branch 5. Upstream control is a discharge vs. time curve for the outflow from Gorge Lake.

5) Skagit River from Newhalem to Bacon Creek - Length of channel is 11 miles. Divided into 22 interior nodes. Downstream control is an equality of discharge and elevation with the upstream exterior node of Branch 6. Upstream control is the discharge vs. time curve of the outflow from Branch 4.

6) Skagit River between Bacon Creek and Rockport - Length equals 15 miles, divided into 14 nodes. Downstream control is an elevation and discharge equality. Upstream control is the discharge vs. time outflow from Branch 5.
7) Skagit River between Rockport and Baker River confluence - Length equals 12 miles, divided into 13 interior nodes. Downstream control is an elevation and discharge equality with Branch 8, upstream control is the time vs. discharge from Branch 6.

8) Skagit River Downstream of Baker Confluence - Length of channel is 37 miles. Divided into 38 interior nodes. Downstream control is an elevation vs. time curve representing the flooding behavior of the Skagit Delta. Upstream control is an equality of elevation and discharge with Branch 7.

3.2.1 Cross Section Computations
Cross sections for Ross, Diablo, and Gorge Lakes, and for the Skagit River upstream of its confluence with the Sauk River were prepared by Centrac Associates, Inc, of Seattle. Centrac performed field surveys and utilized existing detailed contour maps to produce cross sections of the flood plain and reservoirs. Cross sections for the Skagit River downstream of the Sauk were prepared by Hydrocomp as part of the Dam Failure Inundation study of the Baker River Project (1981). These sections were calculated from the Department of Natural Resources, 1 inch=1000 feet scale topographic maps, with some additional detail provided by the Corps of Engineers' 1967 "Flood Plain Information Study" and the 1972 "Flood Insurance Study". As with Centrac's cross sections, these three sources provided good data on the flood plain, but were not as helpful in delineating the detail of actual in-channel cross sections. A fairly uniform low-flow channel shape was assumed, interpolating between the three channel cross sections available on the Skagit. In order to smooth the simulation of the recession limb of the flood hydrographs, this low-flow channel was
added to each cross section. This smoothing of channel cross sections will not affect the accuracy of flood routing results because the low flow channel is full at all times in the simulation and because it represents such a small part of the entire flood flow channel. This smooth low flow channel does however aid the simulation by smoothing the numerical analysis of the recession limb of the flood hydrographs.

3.2.2 Control Structures and Boundary Conditions
The specification of boundary conditions and control structure curves required a number of assumptions. Starting at the upstream end of Branch 1, a constant inflow to Ross Lake of 15,000 cfs was assumed. The FULEQ program requires that water be flowing in each branch at all times. While this inflow value is high, it was necessary in order to provide a high discharge to the Skagit River. The initial discharge is not significant when compared with the magnitude of dam failure flows. The discharge from Ross Dam was also assumed to be 15,000 cfs, to provide a steady state condition. The initial water surface elevation of Ross Lake was assumed to be at the normal full pool level of 1602.5 feet. This corresponds to an initial storage of 1,425,000 acre feet. In calculating the stage versus discharge control structure curves for the failed dams, critical velocity was assumed at the dam sites.

Diablo Dam was assumed to have an initial inflow of 15,000 cfs from Ross Dam, plus an additional 3,000 cfs from Thunder Creek. The lake was assumed to be at full pool with an elevation of 1205 feet and a storage volume of 89,000 acre feet. The dam control structure was assumed to be passing the 18,000 cfs inflow. The spillway gates were not moved during the simulation, although test runs indicated that gate movement would not affect the simulation
results. The control structure curve for the dam was computed based on the "Diablo Dam Spillway Discharge Curve" from the "Ross, Diablo and Gorge Spillway Adequacy Investigation" (1973), and by using a sharp crested weir equation \((C=3.28)\) for overtopping discharges.

The initial inflow to Gorge Lake was assumed to be 20,000 cfs; 18,000 cfs from Diablo and 2,000 cfs of local inflow. The water level was at the normal full pool level of 875 feet. The initial storage was 8500 acre feet. The spillway gates were assumed to be opened 6 feet and were not moved during the simulation. The control structure curve was based on the "Spillway Adequacy Investigation" and a sharp crested weir formula for overtopping discharges. Initial outflow was equal to 20,000 cfs.

In choosing initial discharge values for the reaches of the Skagit River, the intent was to use flows which would be large enough to make the simulation numerically stable and would reduce the effects of unknown variations in the low flow channel. These initial flows improve the stability of the simulation and allow the delineation of maximum probable failure caused flooding. The initial discharge values assumed were:

- Skagit River at Newhalem: 25,000 cfs
- Skagit River at Bacon Creek: 28,000 cfs
- Cascade River at Marblemount: 10,000 cfs
- Sauk River at Skagit River: 22,000 cfs
- Skagit River at Concrete: 62,000 cfs

These initial values represent flows that might be observed once every year or two. Although the initial flows are high, they
represent less than five percent of the peak failure-caused flows and do not affect the accuracy of the results.

At the lower end of the Skagit River, the downstream stage vs. time curve was calculated to simulate the effects of the flooding of the Skagit Delta, up to the height of the eight foot sea dikes. The intricacies of the overland flow runoff pattern and the variable location of river dike failures makes precise simulation of flood wave movement impossible in this delta area.
4.0 FAILURE MODES STUDIED

The three dams of the Skagit Project are modern concrete arch structures, designed and built to survive all foreseeable circumstances. Regular maintenance and inspection are provided to insure that no unsafe conditions develop. Periodic FERC Safety Inspections have revealed no unsafe conditions and no reason to suspect that any portion of the structures or foundations are potential sources of weakness. If a potential weakness had been identified in structural analyses of any of the dams, the failure simulation could hypothesize an initial failure of that weak link, leading to a progressive failure of the entire structure. With no identified site specific hazardous conditions, the failure modes used for this study were formulated based upon historic occurrences of dam failures and the use of engineering judgement.

4.1 Historic Dam Failures

There have been only two major historic concrete arch dam disasters, only one of which resulted in failure. In 1959 the 200 foot high Malpassat Dam in southern France experienced an abutment failure which resulted in the very rapid destruction of more than two-thirds of the dam's structure. The cause of the failure was determined to be weak layers of rock in the foundation of the left abutment. The only early warning was an increase in seepage downstream from this abutment several days before the collapse. The failure of the dam took just a few seconds.

In 1963 the reservoir behind Vaiont Dam experienced a tremendous rockslide which displaced 90% of the 97,000 acre feet of water in the reservoir. The resulting wave overtopped the 869 foot high
thin arch dam by 330 feet. Though subjected to extreme stresses, the dam did not fail. Severe damage and loss of life occurred downstream since most of the reservoir volume was forced out over the dam.

Incidents similar to these which have occurred at concrete gravity dams indicate that whether the cause of the failure is the movement of an abutment or foundation or a failure of some part of the structure, the resulting collapse usually involves most of the dam structure and takes place in less than 10 minutes.

4.2 Specific Project Information

The Draft Report on the 1981 FERC Safety Inspection concluded that all three dams and their foundations were "adequately stable under static and seismic loading conditions". There were determined to be no geologic hazards at the sites or near the reservoirs which would affect the safety of the structures. Maintenance and operations were reported to be satisfactory. This section draws heavily upon that Safety Inspection Report.

4.2.1 Ross Dam

Ross Dam was completed in 1949 to a height of 540 feet. The dam is situated in a glacial canyon carved in Skagit granite-gneiss. The dam has two chute spillways with a design capacity of 127,000 cfs. The dam was designed to be raised beyond its current crest elevation of 1615 feet.

The foundation for Ross Dam is massive, sound granodiorite and gneiss. Minor shear zones were treated during construction and a deep grout curtain was installed.
The slopes above Ross Lake are tree-covered and exhibit no signs of major landslide activity. No seismic activity has been observed in the area since 1965.

4.2.2 Diablo Dam

The 389 foot high Diablo Dam was completed in 1930. The dam's lower sections are sited in a steep granite-gneiss canyon. The upper portion spreads out onto a glacially scoured valley. The predominant foundation rock is Skagit granite-gneiss. Spillways on both abutments have a total capacity of 97,340 cfs.

The 1981 Safety Inspection revealed no hazards from avalanches or rockslides from the hills above Diablo Lake or from earthquake loadings.

4.2.3 Gorge Dam

Gorge Dam is 300 feet high and was completed in 1961. The spillway is on a gravity section of the left abutment and has a capacity of 145,000 cfs. The predominant foundation rock is granite-gneiss. During construction, all weathered rock was removed and a grout curtain and drain system were installed.

Some small snow and rock avalanches have occurred along the banks of Gorge Lake, but no signs of major landslide movement have been observed.
4.3 Modes of Failure - Skagit Project

The failure modes assumed in this study were an attempt to simulate the rapid or instantaneously developing collapse documented in most historic failures of concrete dams. For the simulation, the reservoirs were assumed to be full, with a constant flow through the damsite, when the dam was uniformly removed over a 60 second time span. Flow through the damsite was assumed to be at critical depth throughout the simulation, with the depth of flow at the damsite controlling the discharge.

This mode of failure is most like that of an earthquake-induced or foundation failure type of collapse. The simulation of a slowly developing collapse was not performed since this type of failure was not deemed appropriate for this type of dam. A partial collapse of a dam or dams was not simulated since FERC inundation maps require "worst case" results and because a partial failure of an arch dam is less likely than a failure involving more than 50 percent of the dam structure. Also, during an actual failure emergency, there is no way of knowing that an impending failure will be slowly developing or will only involve part of the structure. Thus any failure must be assumed to be the "worst case" condition.

The inundation maps are not sensitive to the precise mode or timing of failure assumed at the projects. Inundation maps and elapsed times for the floods to reach points in the Skagit Valley are relatively independent of failure modes. For example if a landslide displaces the contents of Ross Lake in 60 seconds, or in 300 seconds, the timing and peak of the flood at Sedro-Woolley are unchanged.
The situation of the three dams in series defined the four possible combinations or modes of failure, namely:

1. Failure of Gorge Dam.
2. Failure of Diablo Dam, which would cause Gorge Dam to fail.
3. Failure of Ross Dam, which would cause Diablo and Gorge to fail.
4. Failure of all three of the dams, simultaneously.

The initial simulation of case 4, a simultaneous failure of all three dams was performed, and the results were found to be nearly indistinguishable from the results of case 3. Therefore the simulation of this failure mode was not continued.

The remaining three failure modes were simulated to conclusion, i.e., until streamflows had returned to their base conditions throughout the Skagit River.
5.0 RESULTS

The simulation of the three separate failure modes using FULEQ was accomplished, and results were obtained for all cases. The failure wave from Ross Dam exceeds 100 feet in height above Newhalem, and 50 feet throughout the entire length of the Skagit River except in the Delta area. Simulated peak water surface elevations are accurate to plus or minus 5 feet, except in steep canyon areas, More precise simulations are not possible due to 1) the presence of small irregularities in channel shape, and 2) inaccuracies introduced through the computer model's use of a uniform cross sectional velocity.

Each of the simulations started with the removal of one of the dams. A very large peak discharge was produced almost instantly. As the water level in the reservoir then began to fall, so too did the discharge through the damsite. The lowering of the water level in the reservoir introduced perturbations of numerical instability into the simulations due to variations in cross section and reservoir bottom slope. By adjusting the modelling time step, and by smoothing the cross sections, it was possible to avoid these perturbations, without significantly affecting the results. These changes affected water surface elevations by less than one foot.

As the reservoirs became nearly empty and as the flood wave moved down the Skagit and finally out into the flat Delta area, instability problems were again encountered. These problems were worked around by artificially introducing a low flow channel into the bottom of the reservoirs, and by adjusting the downstream cross sections and the downstream controlling depth to create a more accurate representation of the Delta's stage versus discharge
relationship. Problems resulting from decreasing slope of the channel bottom were resolved by breaking the Skagit River into five separate branches.

Though these adjustments may affect certain minor aspects of the results slightly, most of their influence is confined to the recession limb of the flood hydrograph (in the case of the reservoir adjustments) or to the far downstream area of the simulation where the complexities of dike failures and overland flow make precise simulation impossible. Despite these difficulties, good simulation results, within the stated accuracy of five feet for peak water surface elevation, were obtained for all three failure modes.

5.1 Gorge Dam Failure Only

The initial elevation of Gorge Lake was 875 feet, corresponding to a storage of 8,500 AF. Discharge from the dam was 20,000 cfs. The dam was totally removed from the site in a 60 second period, beginning at time=0. Within 30 seconds the discharge through the dam site had reached 600,000 cfs. Peak outflow occurred 60 seconds after the failure began, with a discharge of 790,000 cfs. The water level at the dam site fell rapidly for the first 60 seconds, reaching an 830 foot stage. This negative wave propagates upstream rather more slowly, taking almost twenty minutes to lower the water surface four miles upstream.

As the water level at the dam site falls, the discharge also decreases rapidly. Within 20 minutes after dam failure, the discharge has fallen below 100,000 cfs. By this time 90 percent of the reservoir volume has flowed out. The reservoir is
effectively empty, 30 minutes after failure. Figure 5-1 is the discharge vs. time curve for the failure of the Gorge Dam. Figure 5-2 is the stage vs. time curve.

The outflow from the failed Gorge Dam enters the steep canyon of the Upper Skagit River and moves rapidly downstream. The front of the flood wave reaches Newhalem, within ten minutes of the dam failure. The water level at Newhalem rises 18 feet in the first 5 minutes of the wave's passage. The peak discharge at Newhalem following the failure of Gorge Dam is 233,000 cfs. This is significantly higher than the Probable Maximum Flood, and major damage would be expected.

The flood wave continues on down the Skagit, slowing down and decreasing its peak discharge as the channel becomes wider and less steep. The relatively small volume of water in the flood gives it little ability to resist the attenuating effects of routing. The wave reaches Marblemount one and a half hours after the failure and reaches a peak flow of 62,000 cfs at a time=3 hours. The dam-break wave produces only a two-foot increase in stage at Marblemount.

The flood wave continues to dissipate rapidly. The peak flow causes only a one foot rise in water surface elevation as it passes Concrete. The discharge vs. time and water surface elevation vs. time curves for the routing of the Gorge Dam failure wave are shown in Figure 5-3 and 5-4.
FIGURE 5-1

GORGE DAM FAILURE

FLOW IN CFS (x 100000)

TIME AFTER DAM FAILURE IN HOURS
FIGURE 5-2

GORGE DAM FAILURE
WATER SURFACE ELEVATION AT
○ DAMSITE

ELEVATION IN FEET

890
875
860
845
830
815
800
765
770

TIME AFTER DAM FAILURE IN HOURS

0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

Dam Crest Elevation
Figure 5-3

SKAGIT ROUTING: GORGE DAM FAILURE
* FLOW AT MARBLEMOUNT

FLOW IN CFS (X 10000)

0.0 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0
TIME AFTER DAM FAILURE IN HOURS

30
FIGURE 5-4

SKAGIT ROUTING: GORGE DAM FAILURE
WATER SURFACE ELEVATION AT
MARBLE MOUNT

Elevation of center of town

Elevation of top of bank

TIME AFTER DAM FAILURE IN HOURS

ELEVATION IN FEET
5.2 Diablo Dam Failure

The simulation of the failure of Diablo Dam began with the reservoir at an elevation of 1,205 feet and an outflow and inflow of 18,000 cfs. The dam was completely removed from the channel over a 60-second period and 89,000-acre feet of water in storage was allowed to flow out. Within 30 seconds, the outflow has increased to over 400,000 cfs. After 90 seconds, it is over one million cfs. Five minutes after failure, the peak outflow of 2.5 million cfs is reached. The water level at the dam site has fallen to 1,160 feet and the water surface is lowering throughout the reservoir. The outflow remains above a million cfs for almost 60 minutes, by which time nearly 70,000-acre feet of water have flowed out. The complete emptying of Diablo Lake takes about 90 minutes. The outflow hydrograph and stage vs. time curve for the failure of Diablo Dam are shown in Figure 5-5 and 5-6.

The large outflow from Diablo Lake flows directly into the upstream end of Gorge Lake. The wave causes an immediate rise in the stage of the upstream end of the lake, and continues to move down toward the dam. A number of smaller surface saves move through Gorge Lake, but the first major change in stage at Gorge Dam is observed 6 minutes after Diablo's failure. The water level at the dam rises rapidly from its initial 875 feet. The parapet wall at 884 feet is overtopped almost immediately. Nine minutes after Diablo Dam fails, Gorge Dam is over-topped by more than 50 feet, and it is assumed to fail. This failure is simulated in the same manner as was described previously. The dam is completely removed from the site in 60 seconds, beginning at time=9 minutes.

The failure of the overtopped Gorge Dam creates an immediate outflow of 1.5 million cfs. At this time there is still over 2
FIGURE 5-5

DIABLO DAM FAILURE

[Graph showing flow in CFS (x 1000000) over time after dam failure in hours]
DIABLO DAM FAILURE
WATER SURFACE ELEVATION AT
□ DAMSITE

FIGURE 5-6
million cfs entering Gorge Lake from Diablo Lake. The outflow from the Gorge Dam site remains above one million cfs from the failure at time=9 minutes until time=45 minutes. In the first hour following the Diablo Dam failure 85,000 acre feet flow through the Gorge Dam site and down into the Skagit River. Both Diablo and Gorge Lakes are empty two hours after the first failure. The routing of flow through Gorge Lake is shown in Figures 5-7 and 5-8.

The flood wave from the overtopping of Gorge Dam reaches Newhalem 15 minutes after Diablo Dam fails. In another fifteen minutes the discharge has peaked at 1.5 million cfs and the water surface has increased by over 50 feet. The entire river valley is inundated. The wave attenuates as it moves downstream, but the peak flow is still over 500,000 cfs as it passes Marblemount at time= 2.25 hours. The water level goes up 18 feet in this area, and major flooding would be expected, including most of the town. Similar flooding will occur at all of the low lying communities as the flood wave passes.

The wider flood plain between Marblemount and Concrete reduces the peak flow and slows the movement of the wave. The first flooding around Concrete occurs six or seven hours after the dam failures. The peak flow of 190,000 cfs passes Concrete at time= 8 hours, accompanied by an increase in water level of 12 feet. This size flood would cause damage to low lying farms but would not severely affect the town of Concrete.

Farther down towards the Skagit Delta, the dam-break flood wave attenuates to a peak flow of 165,000 cfs as it passes the first major town of Sedro-Woolley. The flood is smaller than the maximum historic flood of 220,000 cfs, but still large enough to
FIGURE 5-7

GORGE OVERTOPPED BY DIABLO FAILURE
WATER SURFACE ELEVATION AT
GORGE DAMSITE

ELEVATION IN FEET

TIME AFTER DAM FAILURE IN HOURS

36
FIGURE 5-8

GORGE OVERTOPPED BY DIABLO DAM FAILURE

* INFLOW
□ OUTFLOW

FLOW IN CFS (x 10,000)

TIME AFTER DAM FAILURE IN HOURS

37
do major damage to the Skagit Delta area. Protective dikes on both sides of the river would be expected to fail in numerous locations. The exact locations of flooding are impossible to predict. The routed hydrograph and stage vs. time curve for the failure of Diablo and then Gorge Dams are shown in Figures 5-9 and 5-10.

5.3 Ross Dam Failure

The simulation of the failure of Ross Dam used a 60 second removal of the dam, like the other simulations. However the size of the dam and the volume of storage in the reservoir made for much more devastating results in terms of flooding. Four minutes after the dam is failed the outflow from the reservoir is 5.5 million cfs. The peak outflow at time= 6 minutes is over 6 million cfs. Even at this high discharge Ross Lake takes a long time to empty. One hour after the dam is failed the reservoir is still two thirds full. The lake is not empty until 6 hours after failure.

The storage volume of Diablo Lake is insufficient to significantly attenuate the Ross Dam failure flood. Five minutes after Ross fails the water elevation at Diablo Dam has already increased from its initial level of 1205 feet to 1220 feet, two feet over the roadway elevation, and 12 feet over the breast wall. Eight minutes after the failure Diablo Dam is being over topped by more than 50 feet, and a 60 second failure is assumed. This high wall of water combined with the high inflows from Ross Lake result in a discharge from Diablo Lake of over 5 million cfs at failure, increasing to 6.1 million cfs at time= 12 minutes. At this time Diablo is a fast flowing river with the inflow from Ross equal to the outflow to Gorge Lake.
FIGURE 5-9

SKAGIT ROUTING: DIABLO DAM FAILURE

X FLOW AT MARBLEMOUNT
* FLOW AT CONCRETE
□ FLOW AT BURLINGTON

FLOW IN CFS (X 10000)

TIME AFTER DAM FAILURE IN HOURS

39
FIGURE 5-10

SKAGIT ROUTING: DIABLO DAM FAILURE
WATER SURFACE ELEVATION AT
- CONCRETE

Elevation of center of town
Elevation of top of bank

TIME AFTER DAM FAILURE IN HOURS
The flood wave entering Gorge Lake overtops Gorge Dam thirteen minutes after the failure of Ross Dam. Gorge Dam is assumed to fail at time=15 minutes, resulting in a peak discharge of 5.5 million cfs as the failure wave passes.

The routing of this huge wave down the Skagit River results in massive flooding. The wave reaches Newhalem twenty minutes after Ross Dam fails, and the stage increases 37 feet in two minutes. Water elevation at Newhalem rises more than 100 feet as the wave peaks with a flow of 5.4 million cfs. The entire valley is inundated as the wave passes downstream. At Marblemount the water level climbs to 369 feet, totally flooding the town and all surrounding areas.

Although the widening of the flood plain below Marblemount reduced the peak flow somewhat, the flood wave has too much volume to prevent the inundation of all of the valley from Gorge Dam to the Puget Sound. The flood reaches Concrete 3.3 hours after failure. Within four more hours the peak flow of over two million cfs passes Concrete, and the entire town is covered by up to 50 feet of water. The wave reaches Sedro-Woolley after six hours and peaks with a flow of 1.7 million cfs, and at an elevation of 76 feet. All dikes are completely overtopped as the flood moves out onto the Skagit Delta. Fifteen hours after the failure of Ross Dam, the Skagit Delta is inundated by between 20 and 50 feet of water. Only the ridges above the 50 foot elevation will not be covered. The routing of the Ross Dam failure wave is shown in Figures 5-11 thru 5-19.
FIGURE 5-11

ROSS DAM FAILURE

OUTFLOW

FLOW IN CFS (X 1000000)

TIME AFTER DAM FAILURE IN HOURS
FIGURE 5-12

ROSS DAM FAILURE
WATER SURFACE ELEVATION AT
DAM SITE

ELEVATION IN FEET

TIME AFTER DAM FAILURE IN HOURS

43
FIGURE 5-13

OVERTOPPING FAILURE OF DIABLO

* INFLOW

■ OUTFLOW

FLOW IN CFS (X 1000000)

0.0 0.75 1.50 2.25 3.00 3.75 4.50 5.25 6.00

TIME AFTER DAM FAILURE IN HOURS

44
FIGURE 5-14

DIABLO OVERTOPPED BY ROSS FAILURE
WATER SURFACE ELEVATION AT
□ DAMSITE

ELEVATION IN FEET

Elevation of roadway

TIME AFTER DAM FAILURE IN HOURS

45
FIGURE 5-15

GORGE OVERTOPPED BY ROSS DAM FAILURE

* INFLOW

□ OUTFLOW

FLOW IN CMS X 1000000

TIME AFTER DAM FAILURE IN HOURS

0.00 0.75 1.50 2.25 3.00 3.75 4.50 5.25 6.00

0.0 0.7 1.5 2.2 3.0 3.7 4.5 5.2 6.0 6.7

46
FIGURE 5-16

GORGE OVERTOPPED BY ROSS FAILURE
WATER SURFACE ELEVATION AT
DAMSITE

ELEVATION IN FEET

TIME AFTER DAM FAILURE IN HOURS

Dam Crest Elevation
FIGURE 5-17

SKAGIT ROUTING: ROSS DAM FAILURE

* FLOW AT MARBLEMOUNT
△ FLOW AT CONCRETE
□ FLOW AT BURLINGTON

FLOW IN CFS ($\times$ 1000000)

TIME AFTER DAM FAILURE IN HOURS
FIGURE 5-19

SKAGIT ROUTING: ROSS DAM FAILURE
WATER SURFACE ELEVATION AT
\( \square \) CONCRETE
FIGURE 5-19

SKAGIT ROUTING: ROSS DAM FAILURE
WATER SURFACE ELEVATION AT
\[
\begin{array}{c}
\square \text{ BURLINGTON}
\end{array}
\]

\begin{figure}
\centering
\includegraphics[width=\textwidth]{skagit_dam_failure.png}
\caption{Water surface elevation at Burlington following the Skagit routing of the Ross Dam failure.}
\end{figure}
6.0 CONCLUSIONS

The three failure modes simulated produced peak discharges and water surface elevations high enough to cause significant damage along the Skagit River. In the case of a single failure of Gorge Dam, the damage would be minor and confined to the small towns above Concrete. A failure of Diablo Dam would cause Gorge Dam to fail by overtopping, and would result in major damage above Concrete. For the failure of Ross Dam, both Diablo and Gorge would fail by overtopping and the damage would be extensive, with the possibility of major inundation and loss of life in the towns of Diablo, Newhalem, Concrete, Sedro-Woolley, and Burlington. Peak flows and elevations for six locations along the Skagit River are listed in Table 6.1, along with the predicted water surface elevations for the 100-year flood from the 1972 Corps of Engineers Flood Insurance Study.

Table 6.2 lists the critical flow levels for six locations, and the length of time from dam failure until these levels are exceeded. These water surface elevations are called "Zero Damage Flows", and indicate the water level above which damage would be expected to start occurring.

The worst flooding was produced by the failure of Ross Dam, followed by the failure of Diablo and Gorge Dams by overtopping. This failure mode was used in preparing the Inundation Map shown in Figure 6-1.
Table 6.1 Peak Flows and Elevations

<table>
<thead>
<tr>
<th>FAILURE MODE</th>
<th>LOCATION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Diablo</td>
</tr>
<tr>
<td>Corps of Engr. Flood Insurance Study</td>
<td>N/A</td>
</tr>
<tr>
<td>Gorge Dam Only</td>
<td>22,000</td>
</tr>
<tr>
<td>Diablo Dam Followed by Overtopping Failure of Gorge Dam</td>
<td>2,530,000</td>
</tr>
<tr>
<td>Ross Dam followed by Overtopping Failures of Diablo and Gorge Dams</td>
<td>5,870,000</td>
</tr>
</tbody>
</table>
Table 6.2  Time to Exceed Zero Damage Flow Level

<table>
<thead>
<tr>
<th>Corps of Engineers Zero Damage Flow Level in Feet above NGVD</th>
<th>Diablo</th>
<th>Newhalem</th>
<th>Marblemount</th>
<th>Concrete</th>
<th>Sedro-Woolley</th>
<th>Mount Vernon</th>
</tr>
</thead>
<tbody>
<tr>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>157.5</td>
<td>46.0</td>
<td>27.6</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>FAILURE MODE</th>
<th>TIME TO EXCEED ZERO DAMAGE FLOW LEVEL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(hrs)</td>
</tr>
<tr>
<td>Gorge Dam Failure</td>
<td>N/A</td>
</tr>
<tr>
<td>Diablo Dam followed by Overtopping Failure of Gorge Dam</td>
<td>0.0</td>
</tr>
<tr>
<td>Ross Dam followed by Overtopping Failures of Diablo and Gorge Dams</td>
<td>0.2</td>
</tr>
</tbody>
</table>
7.0 REFERENCES


APPENDIX A — FULL EQUATIONS THEORY

The term "Full Equations Routing" refers to the prediction of flood wave movement in open channels using the complete forms of the energy and momentum equations. It differs from the more commonly used hydrologic routing techniques which ignore the higher order terms. Full Equations routing was utilized in this study because the failure of a dam or dams causes rapidly changing flood waves such that the absence of these terms significantly affects the accuracy of the predicted wave motion.

The wave motion of interest in this study is restricted to long waves, wherein the wave length is much longer than the depth of water. Other significant assumptions are: one dimensional flow (velocity in longitudinal channel direction only), rigid boundaries, and an essentially uniform velocity distribution across a section perpendicular to the direction of flow.

A-1 Equations

The principles of conservation of mass and momentum are combined with the above assumptions to produce the St. Venant equations describing long wave motion in open channels. In the case of no lateral inflow these equations are:

\[
\frac{\delta V}{\delta x} + VT \frac{\delta y}{\delta x} + TA_{x}^{y} = 0 \quad \text{Eq. 1}
\]

\[
\frac{\delta V}{\delta t} + V \frac{\delta V}{\delta x} + g \frac{\delta y}{\delta x} = g \left( S_{o} - S_{f} \right), \quad \text{Eq. 2}
\]

in which \( S_{f} = \frac{V |V|}{k^2} \). \quad \text{Eq. 3}

A-1
The symbols are defined as:

- $A$ - area of flow perpendicular to the direction of flow,
- $x$ - distance along the channel,
- $y$ - depth of flow normal to the bottom of the channel,
- $t$ - time,
- $V$ - average velocity of flow,
- $T$ - top width of flow in a section,
- $A_x$ - rate of change in area with distance (constant depth),
- $S_f$ - friction slope,
- $S_o$ - bottom slope,
- $K = \frac{1.49}{n} R^{2/3}$ - conveyance-area ratio,
- $R$ - hydraulic radius,
- $n$ - Maning's roughness coefficient, and
- $g$ - acceleration due to gravity.

Yen (1972) and Strelkoff (1969) present derivations of these equations and give detailed discussions of the assumptions. Kinematic routing, Modified Puls routing, and Miskingum routing are all restricted special cases described by these equations.

Equations 1 and 2 can be transformed to characteristic form by multiplying the former by $\left( \frac{x}{C/A} \right)$ and adding it to the latter, multiplied by $g$ (Strelkoff, 1970). This transformation results in the following equations:

\[ \frac{\delta V}{\delta t} + \frac{\delta V}{\delta x} \frac{dx}{dt} + \frac{g}{c} \frac{\delta V}{\delta t} + \frac{\delta V}{\delta x} \frac{dx}{dt} = g (S_o - S_f) + \frac{c}{A} V A_x^g \]

Eq. 4

and

\[ \frac{dx}{dt} = V + c \]

Eq. 5

where $c = $ celerity of a small wave = \( \left( \frac{g A}{T} \right)^{1/2} \).
The bracketed terms in eq. 4 represent the total time rate of change of velocity and depth respectively in the directions given by eq. 5. Equation 4 can be rewritten as:

\[
\frac{dv}{dt} + g \frac{dy}{dt} = g (s_0 - s_f) + \frac{C}{x} v a^y
\]  

Eq. 4a

with the reminder that the derivatives on the left hand side of eq. 4a are along the directions given by eq. 5. Eqs. 4 and 4a are therefore valid along the paths given by eq. 5. These paths define characteristic lines in the \((x,t)\) plane. (See Fig. A-1.) Every point in the \((x,t)\) plane has two characteristic lines passing through it: one line is given by the plus sign in eq. 5 and the other is given by the minus sign in eq. 5. The first family of lines are called the forward characteristics and the second are called the backward characteristics. The forward and backward characteristics are labelled as \(C^+\) and \(C^-\) respectively on Fig. A-1.

The solid \(C^+\) and \(C^-\) lines give the pattern of the characteristic lines through the point \(P\) when the flow is subcritical (the flow is subcritical whenever \(V < c\)) and from left to right (this direction is taken as being positive and also downstream). Rearranging eq. 5 gives the slope of the characteristics as drawn on the \((x,t)\) plane:

\[
\frac{dt}{dx} = \frac{1}{V + c}
\]  

Eq. 6

The slope of the \(C^+\) characteristic (given by the upper sign in eq. 6) is positive except in the rare case of supercritical flow in the upstream direction. On the other hand the slope of the \(C^-\) characteristic is negative except in the more common case of supercritical flow downstream. Therefore the \(C^-\) characteristic shown as a dotted line on Fig. A-1 represents the
position of the C-characteristic for supercritical flow in the downstream direction.

A-2 Solutions

No explicit solution exists for the St. Venant equations. Hence solutions of these equations with appropriate initial and boundary conditions have been developed using numerical techniques. A wide variety of techniques have been studied. Liggett and Woolhiser (1967), Amein and Fang (1970), Strelkoff (1970) and Price (1974) give representative surveys and comparisons of methods. Most methods have been developed and are applicable when the flow is subcritical. Zovne (1970) and Jolly and Yevjevich (1974) studied supercritical flow and found that the specified time interval method of characteristics is applicable. Flow in steep streams and below a failing dam are frequently supercritical. Both conditions are encountered in dam failure studies. Therefore, the specified time interval method of characteristics was selected for streamflow routing in dam failure analyses.

Figure A-1 shows the rectangular grid used for the numerical solution imposed on the \((x,t)\) plane. The values of depth and velocity are known at the grid points along the channel at time \(t_1\). The problem is to predict the values at the grid points (also called nodes), at time \(t_2\). The typical computation element is shown on Figure A-1. Nodes labelled \(R\), \(M\), and \(L\) are at grid points at time \(t_1\) and therefore the depth and velocity are known. Values at node \(P\) are unknown and are to be determined. Segments of the two characteristic lines passing through point \(P\) and also intersecting the line \(t_1\) form the basis for the specified time interval method of characteristics. The points of intersection are labelled \(S\) and \(F\) on Fig. A-1. Algebraic equations giving relationships among variables at points \(P\), \(L\), \(M\), \(R\), \(S\), and \(F\) are developed to approximate the differential relationships given by eqs. 4 and 5.
Figure A-1 Rectangular Computation Grid for Specified Time Interval Method of Characteristics
The solution procedure for these algebraic equations depends on the location of node P along the channel. Node L and S are missing if node P is at the left end of the channel and node R and possibly node F are missing if node P is at the right end of the channel. These end nodes are called boundary nodes. The nodes not on a boundary are called interior nodes. Further discussion is presented in section 3.1.2.

A-2.1 Solution at an Interior Node

The major steps involved in the solution procedure at an interior node are given in step by step manner as follows:

1. Estimate location of the C+ characteristic, i.e. locate point S.
2. Estimate location of the C- characteristic, i.e. locate point F.
3. Estimate the values of velocity and depth at point F using equations defined along the two characteristics.
4. Refine the location of point S using velocity and depth values at point P.
5. Refine the location of point F using velocity and depth values at point P.
6. Refine values of velocity and depth at point P.
7. Repeat steps 4 through 6 until the change in the values for velocity and depth at point P is acceptably small.

This highly implicit iterative solution must be used because the governing equations (eqs 4 and 5) are highly interdependent. For example, point S cannot be located with precision until the velocity is known at point S and
also at Point P. Neither value is known at the start and estimates followed by successive refinements must be used.

A-2.2 Solution at a Boundary Node
The solution procedure depends on the boundary type (upstream or downstream), on the state of flow at the boundary (subcritical or supercritical) and on the conditions known at the boundary.

A-2.2.1 Upstream boundary node — Only the backward characteristic can pass through point P and also intersect line t1 when the flow at the upstream boundary (left end of the channel) is subcritical. The most commonly available boundary condition at the upstream boundary node is a hydrograph giving discharge into the channel as a function of time. The major steps in the solution in this case are:

1. Estimate location of C-characteristic, i.e. locate point F.

2. Estimate value of velocity and depth at point P using the known flowrate at the boundary and the algebraic equation along the C-characteristic.

3. Refine the location of point F using the velocity and depth values at point P.

4. Refine values of velocity and depth at point P.

5. Repeat steps 3 and 4 until the change in the values of velocity and depth is acceptably small.

Figure A-1 and the related discussion in section A-1 show that no characteristic lines passing through point P and also intersecting line t1 exist when the flow at the upstream boundary is supercritical. The boundary
condition must therefore supply both depth and velocity at the boundary. A solution for this case is obtained by neglecting the local and convective acceleration terms in eq. 1 to yield:

\[
\frac{\delta y}{\delta x} = S_o - S_f \quad \text{Eq. 7}
\]

Substitution of the definition of \(S\) in eq. 7 and approximating the partial derivative of depth by the forward difference of depth at time \(t_1\) gives:

\[
V_p = Q_p / A_p = K_p S_o - f \frac{Y_R - Y_M}{\Delta x}^{1/2} \quad \text{Eq. 8}
\]

where \(K_p\) and \(Q_p\) are the conveyance-area ratio and the known boundary discharge respectively at point \(P\), and \(f\) is an adjustment factor for the water surface slope. When \(f = 0\) iterative solution of eq. 8 gives the normal depth and velocity for the known flowrate while \(f > 0\) corrects the normal values for the effect of water surface slope.

A-2.2.2 Downstream boundary node -- the \(C^+\) characteristic is the only characteristic line to intersect line \(t_1\) and also pass through point \(F\) when the flow is subcritical. A control giving a unique relationship between the stage and the discharge or discharge into a body of water with a surface elevation known as a function of time will give useful downstream boundary conditions. Frequently neither situation applies and eq. 8 with the forward difference replaced by the backward difference:

\[
V_p = K_p S_o - f \frac{M - L}{\Delta x}^{1/2} \quad \text{Eq. 9}
\]

gives the downstream boundary condition. The major steps in the solution procedure are the same for each of the above cases:
1. Estimate the location of the C+ characteristic, i.e. locate point S.

2. Estimate values of velocity and depth at point P using the algebraic equation along the C+ characteristic and the appropriate boundary condition.

3. Refine the location of point F using the velocity and depth at point P.

4. Refine values of velocity and depth at point P.

5. Repeat steps 3 and 4 until the change in the value of velocity and depth is acceptably small.

Supercritical flow at the downstream boundary precludes the specification of a boundary condition. The solution can proceed as for an interior node because both characteristic lines passing through point F also intersect the line 11.

A-2.3 Channel Routing Process

The channel routing process is a combination of the above solutions. The initial conditions to start the process are derived from a steady state profile analysis in the channel. The depth and velocity at the end of the current time step can then be computed for each interior node using the solution procedure given in Section A-2.1. Then the upstream and downstream boundary node depths and velocities can be computed using the procedures in section A-2.2. This completes the computations for one time step. The next time step repeats the same set of calculations using the just completed values at the nodes as the known values.